ON EFFECT OF RESISTANCE COMPONENT IN WAVE FILTER ELEMENTS AND PERFORMANCE OF NON-IDEAL FILTER SECTIONS *

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ABSTRACT. The paper relates to study of the effect of resistance component in wave-filter elements based on the theory and operation of " non-ideal " low-pass, high-pass and band-pass filter sections.

Characteristic impedance, attenuation and phase constants and cut-off frequency or frequencies of " non-ideal " sections have been deduced and measured. Comparison between characteristics of " non-ideal " and " ideal " sections has been made.

INTRODUCTION

The filter theory as given elsewhere relates to " ideal filter section," that is, those in which the filter elements are pure reactances; the characteristic impedances in transmission and attenuation bands are therefore pure resistances and pure reactances respectively. Such ideal filter sections, properly terminated should give zero and very large attenuations for frequency components in the transmission and attenuation bands respectively. The sharpness of cut-off at the edge or edges of the transmission band can be very great since the resistance in elements being negligible ' Q ' can be very large.

The wave-filter sections consisting of inductance and capacitance elements, designed and constructed in actual commercial practice for low- and high-frequency communication systems, cannot be regarded as " ideal " for reasons shown below. The resistance component in each *single* element (whether of inductance or capacitance type) is appreciable in majority of cases and cannot be regarded as zero. Further, since both series and shunt arms of a filter section may consist of several elements depending on the type of section chosen, the total effective resistance in series as well as in shunt arm will be large. It is therefore desirable to work out the exact performance of such " non-ideal sections " rather than apply the results developed for ideal filter sections to their cases.

The inductance elements consist of coils wound either on laminated iron, stalloy, silicon steel or on permalloy and mu-metal cores for audio-frequency range. They are wound on finely powdered iron[1] or permalloy[4] dust cores for higher frequencies up to 200 kc/s and on ferrocart[6] cores up to

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1500 kc/s and are air core coils[2,3] (screened) at high radio frequencies. In cases where the d.c. component circulates through the coil (forming an element of a filter section), the dust-core and air-core types (preferably the latter type) are the only ones which have to be used even for low-frequency range. Further, for broad-band linear systems, coils wound on laminated iron or stalloy cores or even on dust and ferrocart cores may not be so suitable as the air-core coils since the permeability of the core material in former types depends both on frequency as well as on amplitude of current through the coil. For a given inductance, the resistance component of coil wound on permalloy dust core is negligible, of coil wound on laminated iron, stalloy or iron dust core is small and that of coil with air core is sufficiently large.

The capacitance elements are invariably paper, mica or air condensers. Paper and mica condensers are generally used for audio-range, whereas mica and air condensers are used for carrier and high-frequency ranges. A condenser should be regarded in general as equivalent to a pure capacitance either connected in series with or shunted by a resistance. The equivalence of a condenser to a pure capacitance with a resistance in series may arise either due to the resistance of leads from plates to binding terminals and of plates, joints and contacts or to "equivalent resistance" due to dielectric absorption. The power-factor in this case = $r\omega C$ (where r = total resistance of leads etc. or "equivalent resistance" referred to above, C = capacitance and $\omega = 2\pi \times$ frequency) which increases with frequency. The equivalent resistance due to dielectric absorption varies as $\frac{1}{\omega < 1}$ at low frequencies and as 1 at high frequencies. The equivalence of condenser to a pure capacitance shunted by a resistance may arise due to leakage of energy from one plate to another over the dielectric surface. The power-factor in this case is $\frac{1}{r\omega C}$ which decreases with frequency. The resistance component r varies as $\frac{1}{\omega}$

The piezo-electric crystal elements (mounted between two electrodes) used in h.f. filter sections are equivalent to series resonant circuits (consisting of inductance, effective resistance and capacitance in series) shunted by a capacitance. Having a negligible effective resistance and very high 'Q' value at high frequencies, these elements alone can be regarded as suitable for ideal filters and therefore will not be considered here.

The present paper relates to study of the effect of resistance component in inductance and capacitance elements based on theory and operation of "non-ideal" low-pass, high-pass and band-pass filter sections and to the comparison between characteristics of non-ideal and ideal sections.

LOW-PASS NON-IDEAL SECTION

Consider a proto-type symmetrical T-section in which the total impedance in series arm $Z_1 = R + j\omega L$ and total impedance in the shunt arm $Z_2 = R' - j/\omega C$.

(*a*) *Characteristic impedance.*—From general equation for T-section,

$$Z_0 = \left[\left(\frac{L}{C} + \frac{R^2}{4} + RR' - \frac{\omega^2 L^2}{4}\right) + j\left(\omega LR' - R/\omega C + \frac{\omega LR}{2}\right)\right]^{\frac{1}{2}} \quad \ldots \quad (1)$$

It will be seen from (1) that Z_0 will be in general an impedance containing both resistance and reactance components for all frequencies except for the two shown in (2*b*) and (2*c*).

Z_0 reduces to the value given for ideal T-type filters when $R = 0$ and $R' = 0$.

When $\omega = \dfrac{2}{\sqrt{LC}}$, $$Z_0 = \left[\left(RR' + \frac{R^2}{4}\right) + j\sqrt{\frac{L}{C}}\left(2R' + \frac{R}{2}\right)\right]^{\frac{1}{2}}. \quad \ldots \quad (2a)$$

When $\omega = \dfrac{1}{\sqrt{LC\left(\dfrac{R'}{R} + \dfrac{1}{2}\right)}}$, $$Z_0 = \left[\frac{L}{C} + \frac{R^2}{4} + RR' - \frac{LR}{2C(R + 2R')}\right]^{\frac{1}{2}}. \quad \ldots \quad (2b)$$

= a pure resistance.

When $\omega = 2\sqrt{\dfrac{1}{LC} + \dfrac{R^2}{4L^2} + \dfrac{RR'}{L^2}}$,

$$Z_0 = \left[j\left\{2L\left(\frac{1}{LC} + \frac{R^2}{4L^2} + \frac{RR'}{L^2}\right)^{\frac{1}{2}}\left(R' + \frac{R}{2}\right) - \frac{R}{2C}\left(\frac{1}{LC} + \frac{R^2}{4L^2} + \frac{RR'}{L^2}\right)^{-\frac{1}{2}}\right\}\right]^{\frac{1}{2}}$$

= a pure resistance. ... (2*c*)

If R_0 and X_0 be the resistance and reactance components of Z_0 in equation (1), then

$$Z_0 = R_0 + jX_0. \quad \ldots \quad (3)$$

From (1) and (3)

$$R_0 = \left[\frac{1}{2}\left\{\sqrt{\left(\frac{L}{C} + \frac{R^2}{4} + RR' - \frac{\omega^2 L^2}{4}\right)^2 + \left(\omega LR' - \frac{R}{\omega C} + \frac{\omega LR}{2}\right)^2} + \left(\frac{R^2}{4} + RR' + \frac{L}{C} - \frac{\omega^2 L^2}{4}\right)\right\}\right]^{\frac{1}{2}} \quad \ldots \quad (4)$$

$$X_0 = \left[\frac{1}{2}\left\{\sqrt{\left(\frac{L}{C} + \frac{R^2}{4} + RR - \frac{\omega^2 L^2}{4}\right)^2 + \left(\omega LR' - \frac{R}{\omega C} + \frac{\omega LR}{2}\right)^2} - \left(\frac{R^2}{4} + RR' + \frac{L}{C} - \frac{\omega^2 L^2}{4}\right)\right\}\right]^{\frac{1}{2}}. \quad \ldots \quad (5)$$

(*b*) *Attenuation and phase constant.*—If the ratio of 'sending-end current' (I_1) to 'receiving-end current' (I_2) when the section is terminated by its proper characteristic impedance be I_1/I_2,

then
$$\frac{I_1}{I_2} = 1 + \frac{R + j\omega L}{2(R' - j/\omega C)} + \frac{R_0 + jX_0}{R' - j/\omega C}. \qquad \dots \quad (6)$$

If the propagation constant 'P' $= \log_e (I_1/I_2) = \alpha + j\beta$ (where α = attenuation constant and β = phase constant), then

$$\alpha = \log_e \left[\left\{ 1 + \frac{RR' - L/C + 2(R_0R' - X_0/\omega C}{2(R'^2 + 1/\omega^2C^2)} \right\}^2 + \left\{ \frac{\omega LR' + R/\omega C + 2(X_0R' + R_0/\omega C}{2(R'^2 + 1/\omega^2C^2)} \right\}^2 \right]^{\frac{1}{2}} \qquad \dots \quad (7)$$

$$\beta = \tan^{-1} \frac{\omega LR' + R/\omega C + 2(X_0R' + R_0/\omega C)}{2R'^2 + 2/\omega^2C^2 + RR' - L/C + 2(R_0R' - X_0/\omega C)}. \qquad \dots \quad (8)$$

(*c*) *Cut-off frequencies and sharpness of cut-off.*—In case of non-ideal sections, the cut-off frequencies cannot be calculated from the relations $Z_1/Z_2 = 0$ and $Z_1/Z_2 = -4$ as in case of ideal sections. The above values of Z_1/Z_2 are the *limiting* values between which the characteristic impedance of the ideal section is pure resistance (that is, they give the limits of transmission band), and consequently are inapplicable to case where the characteristic impedance of the section is in general an impedance having both resistance and reactance components in the transmisssion band.

The calculation of cut-off frequencies in this case is derived from equation (7). $d\alpha/d\omega$ can be determined from (7). Then the real and admissible value of ω for which $d\alpha/d\omega$ is maximum or, say, at least of the order of 40 *db* in 1000 *c/s* gives the cut-off frequency. Graphical determination is more convenient.

Experiments have been carried out on a non-ideal section in which

$$L/2 = 10.575 \text{ mH}\ ;\ R = 20\Omega \text{ at } 800 \sim \text{ to } 28.25\Omega \text{ at } 6500 \sim\ ;$$

$$C = 0.1\mu F, \text{ and } R' = 1.5\Omega \text{ at } 800 \sim \text{ to } 0.2\Omega \text{ at } 6500 \sim.$$

The characteristic impedance and attenuation constants were measured according to standard practice. For measurement of phase constant, an arrangement similar to that used by Messrs. Hinton Rendall and White in their paper have been adopted.[5]

Fig. 1 shows the values of R_0 and X_0 components of Z_0 at various frequencies calculated from (4) and (5), and the measured values of $|Z_0|$ over the same range, for the non-ideal section. It further shows the calculated values of R'_0 and X'_0 components of the characteristic impedance of the same section when the elements are purely reactive (that is, of the ideal section).

Fig. 2 shows the attenuation—frequency characteristics of non-ideal section both measured and calculated from (7) as well as that of the ideal section.

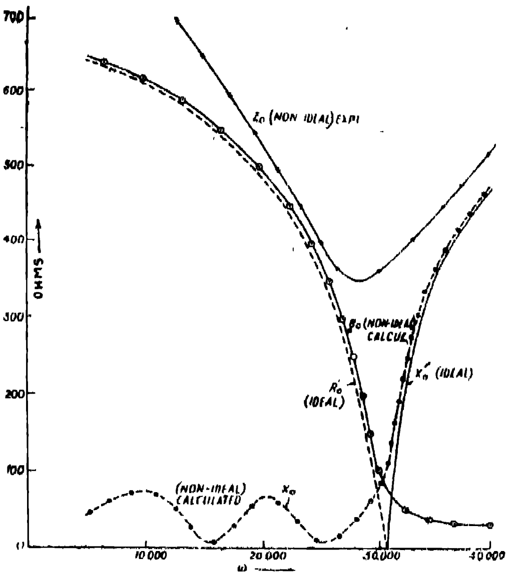

FIG. 1

Fig. 2 also shows the phase-shift—frequency characteristics of non-ideal section both measured and calculated from (8) as well as that of the ideal section.

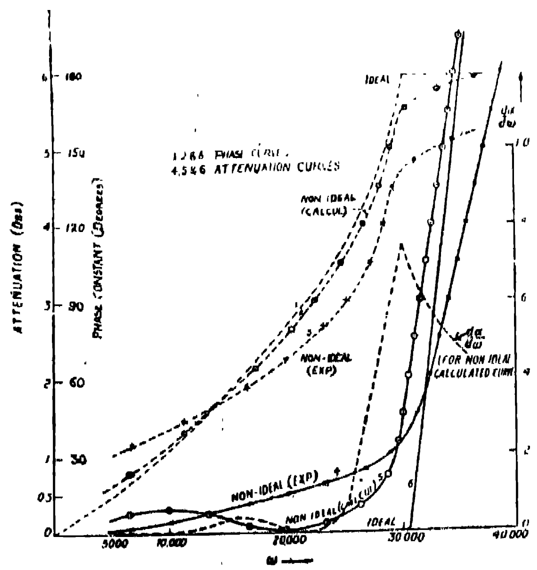

FIG. 2

In Fig. 2, $d\alpha/d\omega$ obtained from the calculated attenuation—frequency characteristic of the non-ideal section is plotted against frequency. From this curve the cut-off frequency of the non-ideal section is found to be 4.8 kc/s.

HIGH-PASS NON-IDEAL SECTION

Consider a proto-type symmetrical T-section, in which the total impedance in the series arm $Z_1 = R' - j/\omega C$ and total impedance in the shunt arm $Z_2 = R + j\omega L$.

(*a*) *Characteristic impedance.*—

$$Z_0 = \left[\left(RR' + \frac{R'^2}{4} + \frac{L}{C} - \frac{1}{4\omega^2 C^2}\right) + j\left(\omega R'L - \frac{R}{\omega C} - \frac{R'}{2\omega C}\right)\right]^{\frac{1}{2}} \qquad \ldots (9)$$

which reduces to the value given for ideal section when $R' = 0$ and $R = 0$.

When $\omega = \dfrac{1}{2\sqrt{LC}}$, $Z_0 = \left[\left(RR' + \dfrac{R'^2}{4}\right) - j\sqrt{\dfrac{L}{C}}\left(\dfrac{R'}{2} + 2R\right)\right]^{\frac{1}{2}}$. $\qquad \ldots (10)$

When $\omega = \sqrt{\dfrac{1}{LC}\left(\dfrac{R}{R'} + \dfrac{1}{2}\right)}$, Z_0 is a pure resistance.

When $\omega = \sqrt{\dfrac{1}{C(4CRR' + CR'^2 + 4L)}}$, Z_0 is a pure reactance.

If R_0 and X_0 are resistance and reactance components respectively of Z_0, then

$$R_0 = \left[\frac{1}{2}\left\{\sqrt{\left(RR' + \frac{R'^2}{4} + \frac{L}{C} - \frac{1}{4\omega^2C^2}\right)^2 + \left(\omega R'L - \frac{R}{\omega C} - \frac{R'}{2\omega C}\right)^2} + \left(RR' + \frac{R'^2}{4} + \frac{L}{C} - \frac{1}{4\omega^2C^2}\right)\right\}\right]^{\frac{1}{2}} \qquad \ldots (11)$$

$$X_0 = \left[\frac{1}{2}\left\{\sqrt{\left(RR' + \frac{R'^2}{4} + \frac{L}{C} - \frac{1}{4\omega^2C^2}\right)^2 + \left(\omega R'L - \frac{R}{\omega C} - \frac{R'}{2\omega C}\right)^2} - \left(RR' + \frac{R'^2}{4} + \frac{L}{C} - \frac{1}{4\omega^2C^2}\right)\right\}\right]^{\frac{1}{2}} \qquad \ldots (12)$$

(*b*) *Attenuation and phase constants.*—

$$\frac{I_1}{I_2} = \left[1 + \frac{RR' - L/C + 2(R'R_0 + X_0\omega L)}{2(R^2 + \omega^2L^2)}\right] - j\left[\frac{\omega R'L + R/\omega C + 2(R_0\omega L - RX_0)}{2(R^2 + \omega^2L^2)}\right]$$

so that
$$\alpha = \log_e \sqrt{A^2 + B^2} \qquad \ldots (13)$$

and
$$\beta = \tan^{-1}\frac{B}{A} \qquad \ldots (14)$$

where
$$A = 1 + \frac{RR' - L/C + 2(R'R_0 + X_0\omega L)}{2(R^2 + \omega^2L^2)}$$

and
$$B = \frac{\omega R'L + R/\omega C + 2(R_0\omega L - RX_0)}{2(R^2 + \omega^2L^2)}.$$

(*c*) *Cut-off frequencies and sharpness of cut off.*—As in the previous section $d\alpha/d\omega$ can be determined from (13), and the real and admissible value of ω for which $d\alpha/d\omega$ is maximum or at least of the order of 40 db in 1000 c.p.s. determines the cut-off frequency.

Experiment have been carried out on a non-ideal section in which

$$2C = 0.3\mu F\ ; \quad \frac{R'}{2} = 10.8\Omega \text{ at } 800\sim \text{to } 0.2 \text{ at } 5000\sim\ ;.$$

$$L = 50mH \text{ and } R = 13.55\Omega \text{ at } 800\sim \text{ to } 18.10\Omega \text{ at } 5000\sim.$$

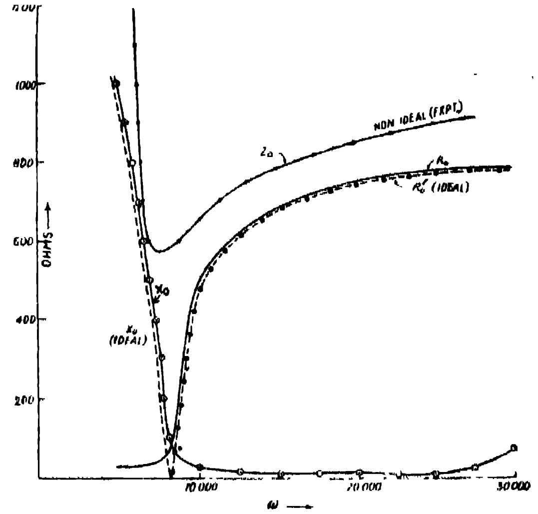

FIG. 3

Fig. 3 shows the values of R_0 and X_0 components of Z_0 calculated from (11) and (12) and the measured value of $|Z_0|$ over the frequency range. The R'_0 and X'_0 components for the ideal section over the same range is also shown.

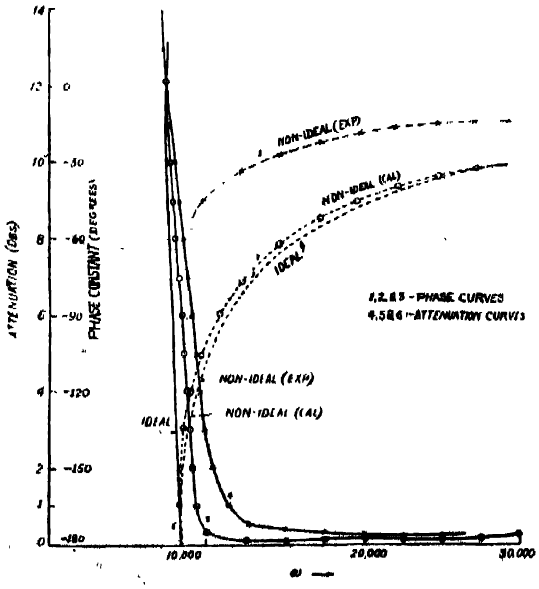

FIG. 4

Fig. 4 shows the attenuation—frequency characteristics of non-ideal section both measured and calculated from (13) as well as that of the ideal section. Fig. 4 also shows the phase-shift—frequency characteristics of non-ideal section both measured and calculated from (14) as well as that of the ideal section.

BAND-PASS NON-IDEAL SECTION

Consider a symmetrical T-section in which the total impedance in the series arm $Z_1 = R_1 + j\left(\omega L_1 - \frac{1}{\omega C_1}\right)$ and total impedance in the shunt arm $Z_2 = R_2 + j\left(\omega L_2 - \frac{1}{\omega C_2}\right)$; R_1 and R_2 include resistances in inductive as well as capacitative elements.

(*a*) *Characteristic impedance.*—

$$Z_0 = \left[\left\{R_1 R_2 - \left(\omega L_1 - \frac{1}{\omega C_1}\right)\left(\omega L_2 - \frac{1}{\omega C_2}\right) + \frac{R_1^2}{4} - \frac{1}{4}\left(\omega L_1 - \frac{1}{\omega C_1}\right)^2\right\}\right.$$
$$\left. + j\left\{R_1\left(\omega L_2 - \frac{1}{\omega C_2}\right) + R_2\left(\omega L_1 - \frac{1}{\omega C_1}\right) + \frac{R_1}{2}\left(\omega L_1 - \frac{1}{\omega C_1}\right)\right\}\right]^{\frac{1}{2}} \quad \ldots \quad (15)$$

which reduces to that of the ideal case when $R_1 = 0$ and $R_2 = 0$.

If R_0 and X_0 are the resistance and reactance components respectively of Z_0, then

$$R_0 = \left[\frac{\sqrt{a^2+b^2} + a}{2}\right]^{\frac{1}{2}} \quad \ldots \quad (16)$$

$$X_0 = \left[\frac{\sqrt{a^2+b^2} - a}{2}\right]^{\frac{1}{2}} \quad \ldots \quad (17)$$

where $a = R_1\left(R_2 + \frac{R_1}{4}\right) - \left(\omega L_1 - \frac{1}{\omega C_1}\right)\left\{\left(\omega L_2 - \frac{1}{\omega C_2}\right) + \frac{1}{4}\left(\omega L_1 - \frac{1}{\omega C_1}\right)\right\}$

and $b = R_1\left(\omega L_2 - \frac{1}{\omega C_2}\right) + \left(R_2 + \frac{R_1}{2}\right)\left(\omega L_1 - \frac{1}{\omega C_1}\right)$.

(*b*) *Attenuation and phase constants.*—

$$\frac{I_1}{I_2} = \left[1 + \frac{R_1 R_2 + \left(\omega L_1 - \frac{1}{\omega C_1}\right)\left(\omega L_2 - \frac{1}{\omega C_2}\right) + 2\left\{R_0 R_2 + X_0\left(\omega L_2 - \frac{1}{\omega C_2}\right)\right\}}{2\left\{R_2^2 + \left(\omega L_2 - \frac{1}{\omega C_2}\right)^2\right\}}\right.$$
$$\left. + j\,\frac{R_2\left(\omega L_1 - \frac{1}{\omega C_1}\right) - R_1\left(\omega L_2 - \frac{1}{\omega C_2}\right) + 2\left\{R_2 X_0 - R_0\left(\omega L_2 - \frac{1}{\omega C_2}\right)\right\}}{2\left\{R_2^2 + \left(\omega L_2 - \frac{1}{\omega C_2}\right)^2\right\}}\right]$$

Then $$\alpha = \log_e \sqrt{p^2 + q^2} \quad \ldots \quad (18)$$

and
$$\beta = \tan^{-1}\frac{q}{p} \qquad \dots \quad (19)$$
where
$$p = 1 + \frac{R_1R_2 + \left(\omega L_1 - \frac{1}{\omega C_1}\right)\left(\omega L_2 - \frac{1}{\omega C_2}\right) + 2\left\{R_0R_2 + X_0\left(\omega L_2 - \frac{1}{\omega C_2}\right)\right\}}{2\left\{R_2^2 + \left(\omega L_2 - \frac{1}{\omega C_2}\right)^2\right\}}$$
$$q = \frac{R_2\left(\omega L_1 - \frac{1}{\omega C_1}\right) - R_1\left(\omega L_2 - \frac{1}{\omega C_2}\right) + 2\left\{R_2X_0 - R_0\left(\omega L_2 - \frac{1}{\omega C_2}\right)\right\}}{2\left\{R_2^2 + \left(\omega L_2 - \frac{1}{\omega C_2}\right)^2\right\}}.$$

(*c*) *Cut-off frequencies.*—As in the previous sections $d\alpha/d\omega$ can be determined from (18), and the real and admissible values of ω for which $d\alpha/d\omega$ is maximum or at least of the order of 40 db in 1000 c.p.s. give the cut-off frequencies of the section.

Experiments have been carried out on a non-ideal section having the following elements :—

$2C_1 = 0.15\mu F$; $L_1/2 = 19.4 mH$; $R_1/2$ (total) $= 19.3\Omega$ at 800~ to 12.1Ω at 6500~ ; $L_2 = 48.6 mH$; $C_2 = 0.0183\mu F$ and R_2(total) $= 254.3\Omega$ at 800~ to 50Ω at 6500~ ;

Fig. 5 shows the values of R_0 and X_0 components of Z_0 calculated from (16) and (17) as well as the measured and calculated values of $|Z_0|$ over the range.

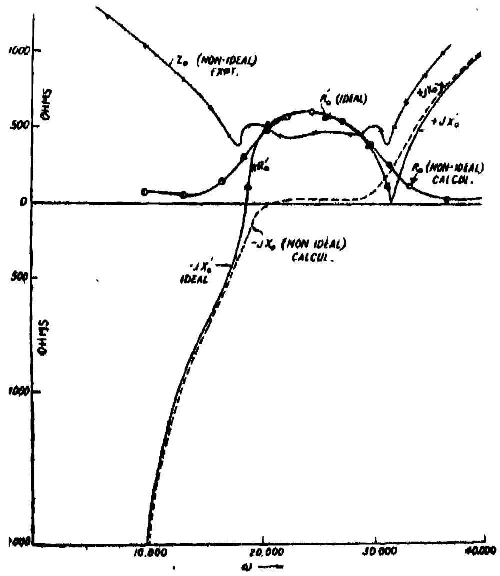

FIG. 5

Fig. 6 shows the attenuation—frequency characteristics of non-ideal section both measured and calculated from (18) as well as that of the ideal section.

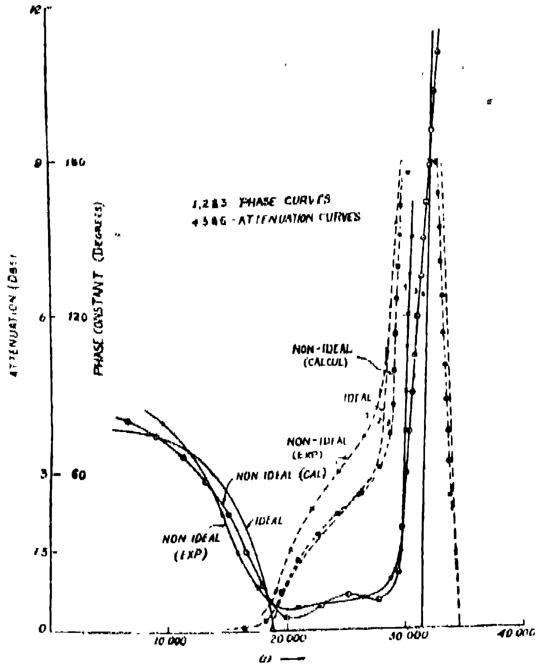

FIG. 6

Fig. 6 also shows the phase-shift—frequency characteristics of non-ideal section both measured and calculated from (19) as well as that of the ideal section.

CONCLUSIONS

The following conclusions have been arrived at :—

(1) The characteristic impedance of "non-ideal" filter section (low-pass, high-pass or band-pass type) will be an impedance containing both resistance and reactance components for frequency components in transmission as well as in attenuation band.

The resistance component of the characteristic impedance within the transmission band is more or less the same in magnitude as the non-reactive characteristic impedance of the ideal section, whereas the reactance component of characteristic impedance in the attenuation band is the same in magnitude as the reactive characteristic impedance of the ideal section in the same range.

(2) The characteristic impedance has been found to be pure resistance and pure reactance only at two particular frequencies respectively.

(3) The attenuation constant within the transmission band varies with frequency and may be from 0.2 to 1.0 db per section with typical values of resistance components in the elements.

(4) The cut-off frequencies of non-ideal sections cannot be obtained from the usual relations $\frac{Z_1}{Z_2} = 0$ and $\frac{Z_1}{Z_2} = -4$ as in the case of ideal filter-sections, since the above relations are the limiting values between which the characteristic impedance of ideal section is pure resistance.

The cut-off frequency or frequencies in this case will be the real and admissible value or values of ω for which $\frac{d\alpha}{d\omega}$ is maximum or at least of the order of 40 db in 1000 c.p.s.

(5) The values of phase constant for ideal and non-ideal sections are observed to be roughly the same.

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